RUWE as an Indicator of Multiplicity

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A short review of recent literature of Gaia Renormalized Unit Weight Error and its sensitivity to multiple systems unresolved in Gaia astrometry.

1. RUWE Definition

The Gaia Renormalized Unit Weight Error (RUWE) is the square root of the normalized chi-square of the astrometric fit to the along-scan (AL) observations, UWE= $[\chi^2/(n-n_p)]^{1/2}$, where n is the number of good CCD observations of the source, and n_p is the number of fit parameters, either 5 [parallax (ϖ), RA (α), Dec (δ), proper motion in RA ($\mu_{\alpha*} = \mu_\alpha \cos\delta$), and Dec ($\mu_\delta$)] or 6 (5 parameters plus pseudo-color for those without high-quality color information) in EDR3 (Lindegren et al. 2021, Sec 3.1). UWE close to 1.0 is expected for well-behaved solutions of single stars. However sources of extreme colors or magnitude can show UWEs larger than 1.0 even if the solution appears reliable, so it is necessary to scale (renormalize) UWE by a factor depending on the magnitude and color of the source; this is the RUWE. RUWE encapsulates all sources of error in the fit to the astrometric model, and is easier to interpret than other indicators such as `astrometric_excess_noise, astrometric_n_bad_obs_al/astrometric_n_obs_al`, or `astrometric_gof_al`. RUWE then is:

$$RUWE = \frac{UWE}{UWE_0(G,C)} \tag{1}$$

where $UWE_0(G,C)$ is the reference UWE value as a function of G magnitude and color of the source. Lindegren (2018) Sec 4 details the derivation of the normalizing factor. RUWE allows establishment of a single value for assessment of the quality of astrometric solution, for distinguishing between "good" and "bad" solutions (Lindegren (2018), Sec 1). RUWE ~ 1.0 for well-behaved single star solutions; typically a threshold of RUWE $\lesssim 1.4$ is used to indicate well-behaved solutions (Lindegren 2018; Lindegren et al. 2021). Sources whose observations are inconsistent with the astrometric 5-parameter model could be caused by binarity (Lindegren 2018, Sec 2) or other factors which cause the photocenter of the source to wobble during the Gaia observation window.

RUWE is related to the chi-squared of the single-source astrometric fit. Equation (1) from Belokurov et al. (2020):

$$RUWE^2 \approx \chi^2 = \frac{1}{\nu}\sum_{i=1}^{N}\frac{R_i^2}{\sigma_i^2} \tag{2}$$

where $\nu = N - 5$ is the number of degrees of freedom, N = `astrometric_n_good_obs_al` which is the number of good observations along scan, 5 is the number of parameters in the DR2 (for which this paper was written, EDR3 includes 6 parameter fits for some sources). R_i, σ_i are model residuals from fit and corresponding centroiding errors for a given observation of a source. Anything that causes the photocenter to wobble – move differently than the center of mass which moves as a single source – such as unresolved subsystems, will increase χ^2 of the fit and thus the RUWE.

2. RUWE and other goodness-of-fit statistics relation to binarity

2.1. The effect of binary separation

The Gaia 5- and 6-parameter solution assumes a single star model for astrometric motion. Despite the fact that a large fraction of stars are in multiple systems, only ∼10% of stars have noticeably non-linear motions over the relatively short time-span covered by the Gaia observation window ($\delta t_{DR2} = 668$ days, $\delta t_{EDR3} = 1038$ days) (Lindegren et al. 2021, Sec. 3.1). Wide multiples may have periods too long to show significant curvature during the observation window; close multiples my have periods so short that deviations in the photocenter position average out over the observation window.

The angular resolution (the minimum separation between sources with different source ids) is $0.18''$ by design, however sources separated by $\lesssim 0.6''$ generally only have 2-parameter solutions in EDR3 (see Lindegren et al. 2021, Sec 5.2 & Fig 6). The resolving power depends on relative magnitude between components, and is $\sim 0.5''$ for equal magnitude stars, up to $\sim 1.2''$ for ΔG = 5 (Gaia Collaboration et al. 2021). However it also depends on the positioning of a given binary on the CCD at the time of the observation. Sources with separation $\gtrsim 1.2''$ can be observed individually, but for separations $<1.2''$ depending on the position angle both stars may fall on the detector simultaneously and cause confusion in the peak finding.

Figure 1 displays a schematic of the Gaia astrometric CCD from Prusti et al. 2016, Fig 5 illustrating how sources are registered. Point spread functions of sources are read in the along-scan (AL) direction to the parallel summing well, which translates the PSF into a line-spread function (LSF). If more than one source falls on the detector, multiple peaks will be present in the LSF, and the amount of blending of those peaks depends on the position angle of the two sources relative to the across-scan (AC) direction and their relative magnitudes.

This is demonstrated in my terrible drawing in Figure 2 illustrating how the LSF is developed from the PSF of a binary with separation small enough that both stars fall onto the detector. If the position angle is something like 30° with respect to the AC direction at the first observation (t_1), and at some time later (t_2) the pair is observed with PA something like 80°, the photocenter of the LSF will shift from t_1 to t_2, especially if one component is fainter than the other. Kervella et al. (2022) coin the term "Gaiacenter" for the epoch pointing of Gaia (after "Hippacenter" for

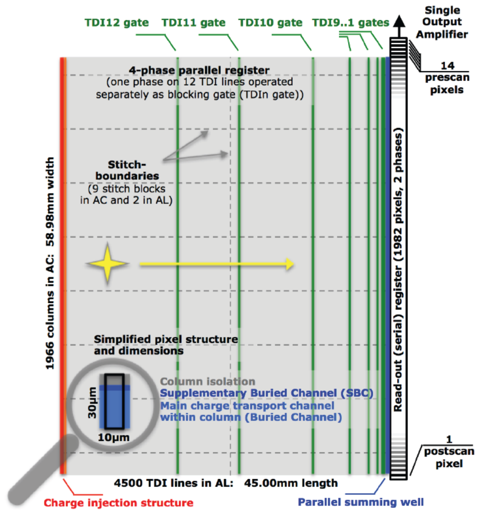

Fig. 1.— Figure 5 from Prusti et al. (2016), displaying a schematic of the Gaia focal plane CCD. The along-scan and across-scan directions are indicated in the top left corner. Stars move across the focal plane from left to right in the diagram, illustrated by the yellow star and arrow, to the summing well and transfer gate. This translates the PSF into a Line Spread Function (LSF). If more than one source falls on the detector, multiple peaks will be present in the LSF, and the amount of blending of those peaks depends on the position angle of the two sources relative to the along-scan direction.

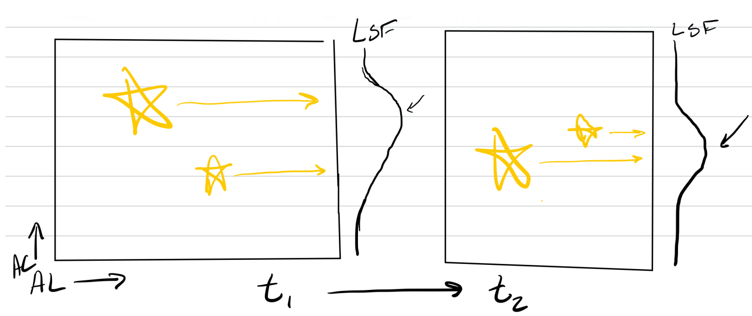

Fig. 2.— My very terrible drawing of the effect of changing position angle on the resulting LSF. If the position angle is something like 30° wrt AC direction at the first observation (t_1), and at some time later (t_2) the pair is observed with PA something like 100°, the photocenter of the LSF will shift from t_1 to t_2, especially if one component is fainter than the other.

Hipparcos observations of double stars from Martin et al. 1997). For binaries from $0.1''$-$1.2''$, the "Gaiacenter" will be closer to the primary as a function of the two stars' relative magnitude, and will vary as a function of position angle on the detector. Thus the astrometric solution will be perturbed, and we should expect higher values of RUWE and significant excess astrometric error.

2.2. Other Goodness-of-fit Statistics to test for binarity

RUWE might be influenced by other factors as well, so some additional Gaia quantities can be useful for testing for unresolved binarity. Prior to the astrometric solution, the data passes the image parameter determination (IPD) stage, where the standard stellar model is fit to the image locations. Goodness-of-fit (gof) statistics from the IPD stage are sensitive to deviations from the simple single point source model, as well as modelling and calibration errors.

`ipd_gof_harmonic_amplitude` measures the amplitude of the variation of goodness of fit as a function of the position angle of the scan direction. It is normally small but can become large for elongated PSF images such as unresolved binaries. `ipd_frac_multi_peak` is the percentage of windows used for astrometric processing that contained more than one peak, so it is sensitive to resolved binaries that produce multiple peaks in the window in some scan directions. `astrometric_excess_noise` quantifies how much motion of the image center deviates from the standard astrometric model in angular units (mas) per AL observation, while the `astrometric_excess_noise_sig` gives the S/N of the excess noise; `astrometric_excess_noise_sig` $\lesssim 2$ is considered insignificant (essentially zero).
`astrometric_excess_noise`/`astrometric_excess_noise_sig` may be used as an estimate of the uncertainty in the excess noise source. However, Belokurov et al. (2020) note that for their subsample of Gaia DR2 solutions, only $\approx$ half of sources with RUWE>1.1 have excess noise > 0, that excess noise "saturates" to zero, and that it is susceptible to systematic trends as a function of color and magnitude; thus RUWE is a better metric to test for binarity than excess noise. (This entire subsection adapted from Lindegren et al. 2021, Sec 5.3)

2.3. Interpreting RUWE value

RUWE = 1.4 is typically used as a cutoff threshold for "good" vs "bad", but it's hardly a strict line in reality. Maíz Apellániz et al. (2021), in their validation of EDR3 using globular clusters, found that the distribution of parallaxes for sources within 1.4<RUWE<2.0 had an average normalized parallax average of 0, and standard deviation < 4, and so were generally safe to use after introducing an additional uncertainty term. RUWE>2 deviated from a normalized average of zero and so have larger biases.

Belokurov et al. (2020) conducted a detailed study of RUWE relation to unresolved binary systems in Gaia DR2 for RUWE <2. Figure 3 shows Figure 1 from their paper, an HR diagram

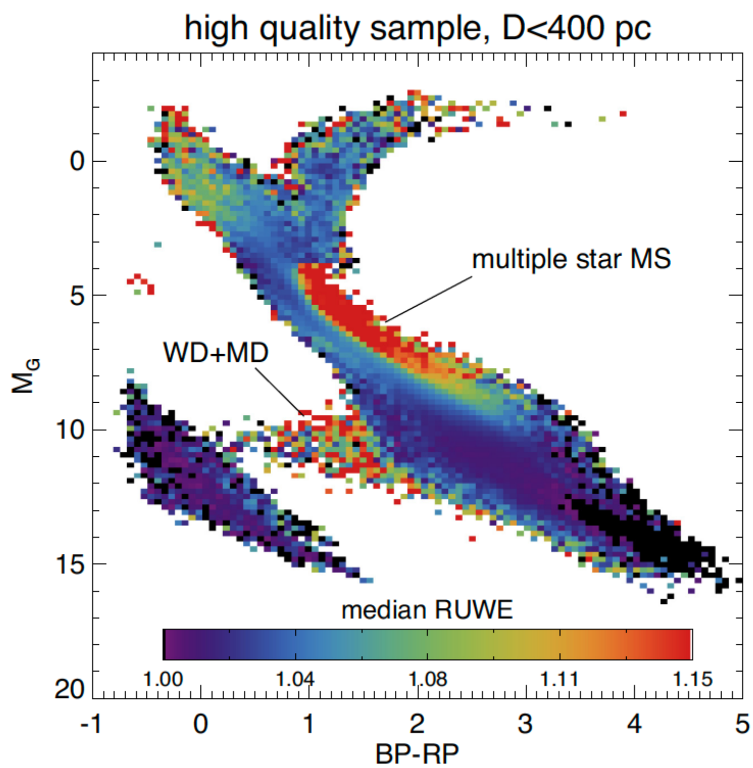

Fig. 3.— Figure 1 of Belokurov et al. 2020 showing an HRD for 4M Gaia DR2 objects from their selection criteria (see Sec 2.1) color coded by RUWE. Two distinct regions of elevated RUWE are evident, corresponding to main sequence (MS) multiples and white dwarf - M dwarf binaries (WD+MD). The supports the claim that RUWE can be used to probe multiplicity.

of Gaia DR2 sources from their selection criteria (Sec 2.1), color-coded by RUWE. Two distinct regions of elevated RUWE are evident, corresponding to main sequence (MS) multiples and white dwarf - M dwarf binaries (WD+MD), evidence for a relation between multiplicity and RUWE. They used the known binary systems of the SB9 spectroscopic binary catalog (Pourbaix et al. 2004) to test RUWE and photocenter wobble correlation.

They parameterized the amplitude of the photocenter perturbation as

$$R_i = R_i^{ss} + \delta\theta_i \tag{3}$$

where $\delta\theta$ is perturbation in arcsec. Converting to perturbation in physical units:

$$\frac{\delta a}{AU} = \frac{\delta\theta}{mas}\frac{D}{kpc} \tag{4}$$

Figure 4 displays a selection of plots from Figure 4 of Belokurov et al. (2020) showing how these perturbations relate to RUWE, distance, and period. They determine that, as predicted, the angular centroid wobble decreases inversely with distance, and that Gaia is sensitive to perturbations

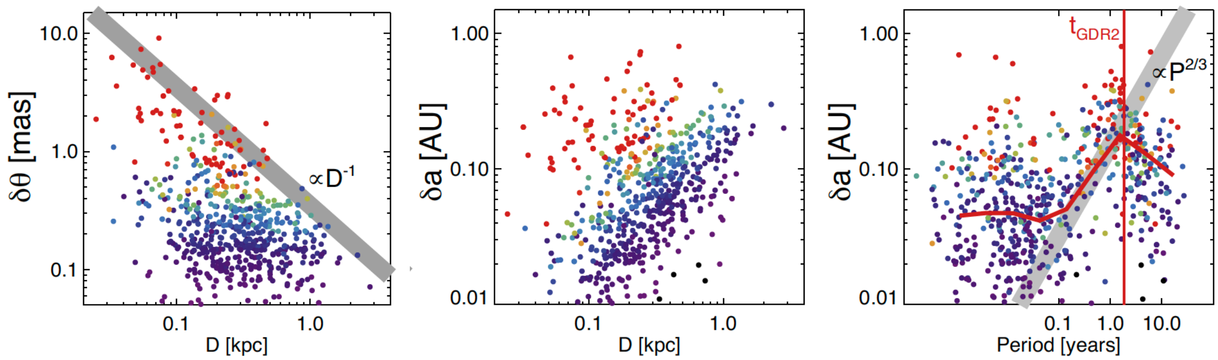

Fig. 4.— Selected plots from Figure 4 of Belokurov et al. 2020 relating perturbation to RUWE. Left: angular displacement $\delta\theta$ as a function of distance - the amplitude of the perturbation drops proportionately to distance. Center: Physical perturbation as a function of distance - systems of the same physical separation a induce perturbation of decreasing amplitude with increasing distance, limiting Gaia's sensitivity. Right: δa as a function of period in years - three regimes: for periods <1 month the photocenter wobble is too small to detect, for 1<P<22 months $\delta a \propto P^{2/3}$ a la Kepler's 3rd law, and for P>22 months only a fraction of the perturbation is detected by Gaia. This sets a regime for which Gaia is sensitive to photocenter wobble.

for sources less than 2-3 kpc distant. Gaia's sensitivity is a function of mass and luminosity ratios. They estimate that for systems less than 1-2 kpc, systems with semi-major axis between 0.1 - 10 AU can be detected. Wider binaries do not produce significant RUWE excess because the centroid perturbation is quasi-linear and absorbed into the proper motion; these can still be detected through the proper motion anomaly (PMa; see Kervella et al. 2022; Brandt 2021 for PMa between Hipparcos and Gaia epochs). Their binary fractions, while limited in ability, are not too far off from those in literature for SB9 and WD binary samples they compare to, indicating that **RUWE can be a reliable indication of the presence of an unresolved companion.**

Similarly, Stassun & Torres (2021) also found strong evidence of correlation between RUWE and unresolved subsystems, even for RUWE values <1.4. They used a sample of benchmark eclipsing binaries (EBs) to probe the mean offset of Gaia EDR3 parallaxes from the benchmark sample, using RUWE as their primary astrometric goodness-of-fit indicator. Figure 5 (left) displays Figure 1 (bottom) from their paper showing the absolute fractional parallax difference between Gaia and EB parallaxes as a function of logRUWE, with systems with known subsystems highlighted in blue halos. The largest parallax difference and largest RUWE values (top right) are entirely populated by these systems. They report a high probability of correlation between parallax difference and RUWE using statistical tests.

The red dashed line (added for this work) marks the nominal "good" value of RUWE = 1.4, below which the correlation persists. Stassun & Torres (2021) find that even below RUWE = 1.4, the RUWE values are very strongly correlated with photocenter motion. Figure 5 (right) displays Figure 3 of their paper showing this correlation for RUWE<1.4. They determine a strong

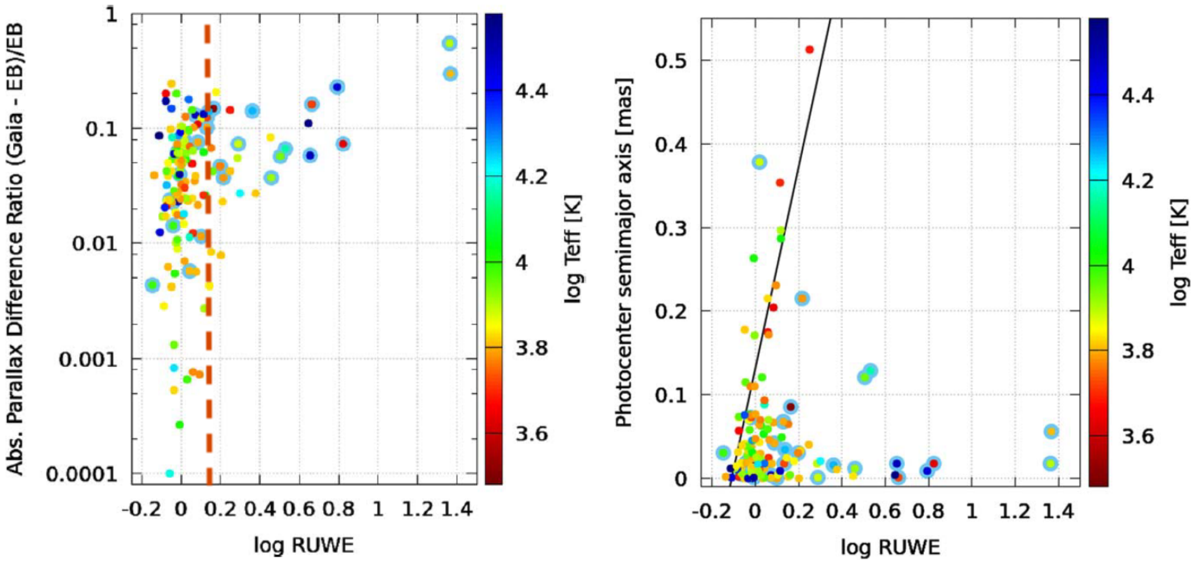

Fig. 5.— Left: Figure 1 (bottom) from Stassun & Torres 2021 showing the absolute fractional parallax difference between Gaia and EB parallaxes on the y-axis vs logRUWE. Blue halos mark EBs with known tertiary companions. The largest parallax difference and largest RUWE values (top right) are entirely populated by these systems. The red dashed line (added for this work) marks the nominal "good" value of RUWE = 1.4. Right: Figure 3 from Stassun & Torres 2021 showing the same data as Figure 1, plotted as a function of the amount of angular photocenter shift ($a''_{\rm phot}$), with the relation of Equation 5 plotted in black line.

correlation ($r^2 = 0.82$) for 1.0$\lesssim$ RUWE $\lesssim$1.4 (which may extend out to RUWE $\sim$ 1.8) of

$$a_{\rm phot}[{\rm mas}] = 1.204 \times \log_{10} {\rm RUWE} + 0.13 \tag{5}$$

shown by the black line in Figure 5 (right). This shows that RUWE is highly sensitive to unresolved companions and **strongly correlated with photocenter motion**, even within the "good" range of 1.0–1.4, and can actually serve as a quantitative predictor of motion.

Penoyre et al. (2020) sought to quantify the effect of photocenter wobble of a single source through numerical modeling. The motion induced by the companion increases the χ^2 of the measurement by a factor of $\chi^2_{\rm binary} = \delta\theta^2/\sigma^2_{\rm ast}$, where $\delta\theta$ is the angular perturbation and $\sigma^2_{\rm ast}$ is the astrometric scatter in the measurement. Then, UWE predicted from these quantities is

$${\rm UWE}_{\rm pred} = \sqrt{\frac{\chi^2_{\rm total}}{N_{\rm obs} - 5}} \simeq \sqrt{1 + \left(\frac{\delta\theta}{\sigma_{\rm ast}}\right)^2} \tag{6}$$

They generated a set of mock observations and determined the resulting effect on the observed RUWE compared to predicted. Figure 6, left, shows a selected plot from their Figure 2 of one of their simulated binary systems. The center of mass (black line) moves as a single body, the center of light (red, simulated data shown as red dots) deviates due to the binary orbit, and the best fitting curve is shown as blue dashed line. The ellipse traced by the center of light is shown on the same scale in the upper left corner.

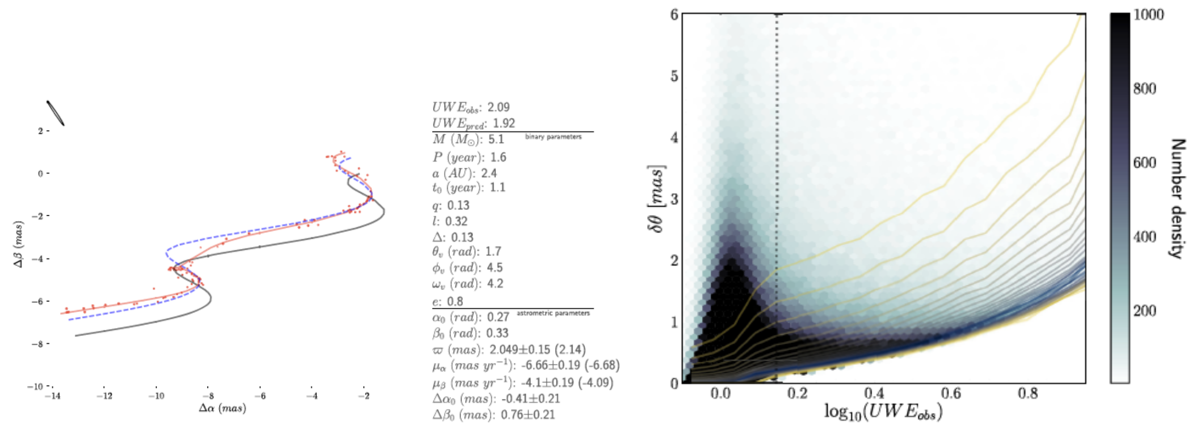

Fig. 6.— Left: Selected plot from Figure 2 of Penoyre et al. 2020 showing one simulated binary system from their numerical model of an unresolved binary. The center of mass (black line) moves as a single body, the center of light (red, simulated data shown as red dots) deviates due to the binary orbit, and the best fitting curve is shown as blue dashed line. The ellipse traced by the center of light is shown on the same scale in the upper left corner. $\Delta\alpha$ and $\Delta\beta$ are projected planet coordinates. Right: Figure 6 from their paper comparing observed UWE to the true angular perturbation due to the binary. The dotted line indicates UWE = 1.4.

They conclude that for shorter period binaries (where period is less than the observational time baseline), the photocenter motion gives increased error, which provides a lower limit to the on-sky angular separation of the binary (there is the possibility that some of the binary motion is absorbed into the center-of-mass motion errors). However it might be difficult or impossible to establish this for any one given system. Periods significantly longer than observational baseline will just cause a constant offset.

High RUWE values have been shown to also explain relative velocity vectors for wide stellar binaries that appear to exceed escape velocity, apparent non-Newtonian motion, which has been used as a test of alternative gravity theories. Clarke (2020) and Belokurov et al. (2020) showed that contamination by sources with unresolved subsystems, and resulting high RUWE values, reproduce the high-velocity tail exceeding escape velocity observed by Pittordis & Sutherland (2019).

2.4. Other causes of RUWE excess

Marginally resolved sources, where stellar image is perturbed from a single PSF but Gaia identifies it as a single source, creates a large centroiding error. The center oscillates wildly with time (as illustrated in Figure 2). Belokurov et al. (2020) shows extremely high values of RUWE

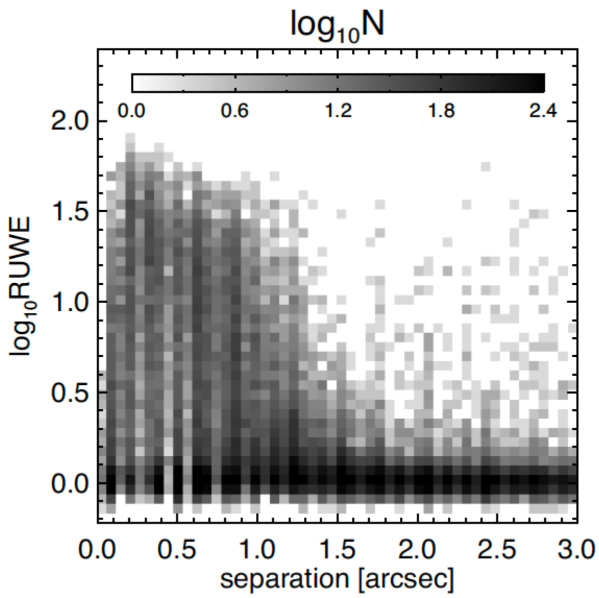

Fig. 7.— Selected plot from Figure 5 of Belokurov et al. 2020 relating RUWE to separation of binaries in Washington Double Star Catalog. Extremely high RUWE values are common for binaries with separation less than 1.5″.

for known visual binaries with separation ρ <1.5″, reproduced here in Figure 7. However there is no correlation between RUWE and ρ — at small separations even faint companions can cause significant centroid displacement. They find that BP/RP excess points to brighter companions and high variability, and that the bulk of semi-resolved double stars could be filtered out by applying cuts to those parameters (see Figure 5 of their paper).

They also report a correlation between RUWE and variability with RR Lyrae, Cepheids, and long period variable sources, particularly for the brightest (5<G<14) RR Lyrae stars, while there is little correlation for fainter stars (G>14). This correlation arises from the normalization of RUWE on these stars. The variable object will be measured at a range of magnitudes and colors and thus can't be normalized using a single value of u_o.

3. Conclusion

RUWE is a robust metric for testing for unresolved multiplicity of Gaia sources. It is highly sensitive to deviations to photocenter motion from center-of-mass motion, even (or especially) below RUWE = 1.4, the often-used cutoff for reliable Gaia astrometry. Several other goodness-of-fit metrics can be used to help probe for binarity, but may not be as strongly correlated as RUWE. RUWE $\gtrsim$2 is not strongly correlated with binarity. The amount of perturbation also establishes a lower limit for the angular separation of the pair; Gaia is likely most sensitive to separations between $0.18''$ and $1.2''$. For RUWE $\lesssim 1.4$, the angular semi-major axis of the photocenter wobble can be estimated via Eqn (5), but should be interpreted with caution for any singular system. RUWE excess can be caused by other perturbations such as marginally resolved systems or chance alignments and variability.

REFERENCES

Belokurov, V., et al. 2020, Monthly Notices of the Royal Astronomical Society, 496, 1922

Brandt, T. D. 2021, ApJS, 254, 42

Clarke, C. J. 2020, MNRAS, 491, L72

Gaia Collaboration et al. 2021, A&A, 649, A6

Kervella, P., Arenou, F., & Thévenin, F. 2022, Astronomy & Astrophysics, Volume 657, id.A7, 26 pp., 657, A7

Lindegren, L. 2018, Re-normalising the astrometric chi-square in Gaia DR2, gAIA-C3-TN-LU-LL-124

Lindegren, L., et al. 2021, Astronomy & Astrophysics, Volume 649, id.A2, 35 pp., 649, A2

Martin, C., Mignard, F., & Froeschle, M. 1997, A&AS, 122, 571

Maíz Apellániz, J., Pantaleoni González, M., & Barbá, R. H. 2021, Astronomy & Astrophysics, Volume 649, id.A13, 10 pp., 649, A13

Penoyre, Z., Belokurov, V., Wyn Evans, N., Everall, A., & Koposov, S. E. 2020, MNRAS, 495, 321

Pittordis, C., & Sutherland, W. 2019, MNRAS, 488, 4740

Pourbaix, D., et al. 2004, A&A, 424, 727

Prusti, T., et al. 2016, Astronomy & Astrophysics, 595

Stassun, K. G., & Torres, G. 2021, The Astrophysical Journal, 907, L33, aDS Bibcode: 2021ApJ...907L..33S

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